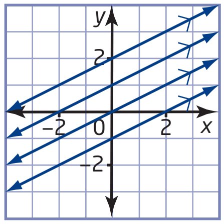
6.5 Find an Equation for a Line Given the Slope and a Point

The slope of a line gives its direction. For any given slope value, there are many possible lines. This family of lines has a slope of $\frac{1}{2}$. How many different lines share the same slope? How do you know? What additional information would you need in order to pinpoint a specific line?



Suppose that you know that a line with a slope of $\frac{1}{2}$ passes through the point (1, 5). There is only one line that does this. In fact, if you know the slope and any point on a line, you can identify its equation.

Example 1 Find the Equation of a Line Given Its Slope and a Point

a) Find the equation of a line with a slope of $\frac{1}{2}$ that passes through (1, 5).

b) Graph the line.

Solution

a) Substitute $x = 1$, $y = 5$, and $m = \frac{1}{2}$ into the slope and y-intercept form of the equation of a line, and solve for b.

$$
\begin{aligned}
y &= mx + b \\
5 &= \frac{1}{2}(1) + b \\
5 &= \frac{1}{2} + b \\
5 - \frac{1}{2} &= b \\
4\frac{1}{2} &= b
\end{aligned}
$$

I can write the equation of a line once I know its slope and y-intercept.

I'm given the slope, so $m = \frac{1}{2}$. I don't know the y-intercept, but I'm given the point (1, 5). So, I know that when $x = 1$, $y = 5$.

The y-intercept is $4\frac{1}{2}$ or $\frac{9}{2}$.

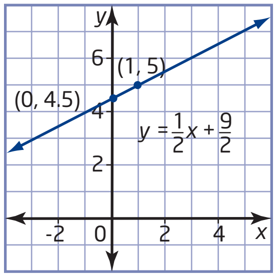
Substitute the values of m and b into $y = mx + b$ to write the equation of the line.

$y = \frac{1}{2}x + \frac{9}{2}$

The equation of the line is $y = \frac{1}{2}x + \frac{9}{2}$.

b) The y-intercept is $\frac{9}{2}$, or 4.5.

Plot this point and the given point (1, 5) to graph the line.



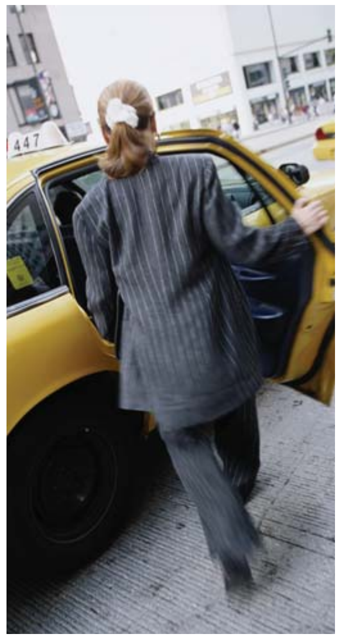
Example 2 Find the Equation of a Partial Variation

Gina knows that it costs \$25 to take a taxi to work, which is 10 km from her home. She forgets what the fixed cost is, but remembers that the variable cost is \$2/km. Her friend lives 12 km from Gina's home. Gina has \$60 to spend on the weekend. Can she afford a round trip to see her friend?

a) Find the fixed cost and write the equation that relates the cost, in dollars, of a trip to the distance, in kilometres.

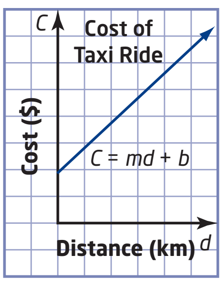
b) Graph the linear relation.

c) Find the cost of a 12-km trip. Can Gina, who has \$60 to spend, afford a round trip of this distance?



Solution

a) This is an example of a partial variation. A graph of cost, C, in dollars, versus distance, d, in kilometres, will produce a straight line.



The variable cost is \$2/km, which represents the slope of the line. The fixed cost is unknown, but it is equal to the vertical intercept. You also know that (10, 25) is on the line.

To find the fixed cost, substitute $d = 10$, $C = 25$, and $m = 2$ into $C = md + b$ and solve for b.

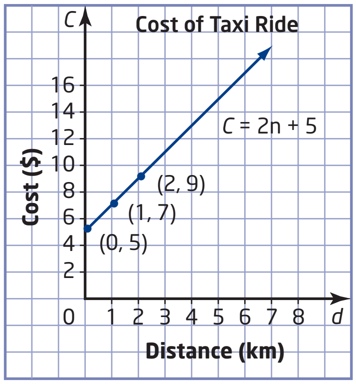
$$\begin{aligned} C &= md + b \\ 25 &= 2(10) + b \\ 25 &= 20 + b \\ 25 - 20 &= b \\ 5 &= b \end{aligned}$$

The vertical intercept is 5. This means that the fixed cost is \$5. To write the equation of the line, substitute m and b into $C = md + b$.

$C = 2n + 5$

The equation $C = 2n + 5$ gives the cost, C, in dollars, for a trip d kilometres long.

b) You can use the vertical intercept and the slope to graph this relation. Plot the point (0, 5). Then, go up 2 and right 1 to find other points.

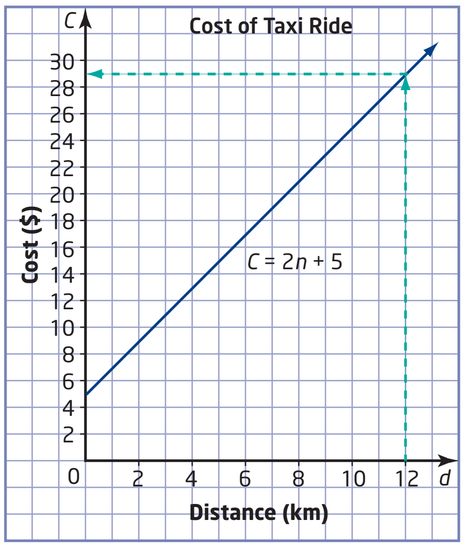


c) To find the cost of a 12-km trip, you can use the graph or the equation.

Method 1: Use the Graph

Extend the graph until you can read the value of C when $d = 12$.

The cost of a 12-km trip is \$29.



Method 2: Use the Equation

Substitute $d = 12$ into the equation relating cost and distance, and solve for C.

$$\begin{aligned} C &= 2n + 5 \\ &= 2(12) + 5 \\ &= 24 + 5 \\ &= 29 \end{aligned}$$

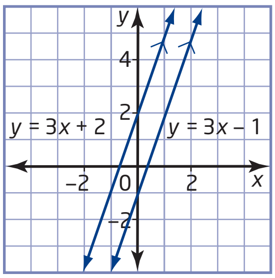
The cost of a 12-km trip is \$29.

A round trip would cost $2 \times \$29$, or \$58. Since Gina has \$60 to spend, she can afford to see her friend.

Sometimes the properties of parallel and perpendicular lines are useful in finding the equation of a line.

Recall from Section 6.4 that parallel lines have the same slope.

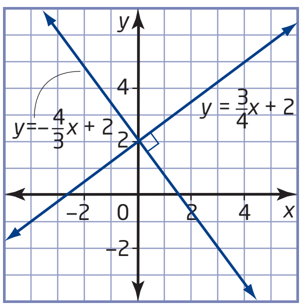
For example, $y = 3x + 2$ and $y = 3x - 1$ are parallel lines. In both cases, $m = 3$.



Perpendicular lines have slopes that are negative reciprocals. The product of the slopes of perpendicular lines equals -1.

For example, $y = \frac{3}{4}x + 2$ and $y = -\frac{4}{3}x + 2$ are perpendicular lines.

$$\left(\frac{3}{4}\right) \times \left(-\frac{4}{3}\right) = -1$$



Literacy Connections

Negative reciprocals are pairs of numbers that are related in two ways. The negative part means that they are opposite in sign. The reciprocal part means that, when expressed as a proper or improper fraction, the numerator of one is the denominator of the other, and vice versa.

For example:

$\frac{5}{3}$ and $-\frac{3}{5}$ 1 and -1

$\frac{1}{2}$ and -2 $\frac{2}{3}$ and -1.5

Example 3 Find Equations of Lines Parallel or Perpendicular to Given Lines

Find the equation of a line

a) parallel to $x - y - 12 = 0$ that passes through $(2, -5)$

b) perpendicular to $y = 4x + 5$ that passes through the origin

Solution

a) The unknown line is parallel to $x - y - 12 = 0$, so it must have the same slope as this line. To find the slope, rearrange the equation to express it in slope y-intercept form.

$$x - y - 12 = 0 \quad \text{Add } y \text{ to both sides.}$$
$$x - 12 = y$$
$$\text{or } y = x - 12$$

The slope of this line, and any line parallel to it, is 1. Substitute $m = 1$ and the known point, $(2, -5)$, into $y = mx + b$ and solve for b.

$$y = mx + b$$
$$-5 = 1(2) + b$$
$$-5 = 2 + b$$
$$-5 - 2 = b$$
$$-7 = b$$

Substitute m and b into $y = mx + b$ to write the equation of the line.
$y = 1x + (-7)$

The equation of the line is $y = x - 7$.

b) The unknown line is perpendicular to $y = 4x + 5$. That means that their slopes are negative reciprocals.

slope of given line: $m = 4$ or $\frac{4}{1}$

negative reciprocal: $-\frac{1}{4}$

The slope of the unknown line is $-\frac{1}{4}$. Use this to find the equation of the line.

The unknown line passes through the origin, which means that its y-intercept is 0. Substitute $m = -\frac{1}{4}$ and $b = 0$ into $y = mx + b$.

$$y = -\frac{1}{4}x + 0$$

$$y = -\frac{1}{4}x$$

The equation of the line is $y = -\frac{1}{4}x$.

Key Concepts

- You can find the equation of a line if you know its slope and one point on the line.
  - Substitute the given slope for m and the coordinates of the given point into the equation $y = mx + b$ and solve for b.
  - Write the equation by substituting the values for m and b into $y = mx + b$.

Communicate Your Understanding

C1 A line has a slope of 3 and passes through the point (2, 1). Explain each step in finding the equation of this line.

Step	*Explanation*
$y = mx + b$	**Start with the slope *y*-intercept form of the equation of a line.**
$1 = 3(2) + b$	
$1 = 6 + b$	
$1 - 6 = b$	
$-5 = b$	

The equation of the line is $y = 3x - 5$.

C2 What is the slope of a line that is perpendicular to a line with each slope?

a) $\frac{3}{5}$ **b)** $-\frac{1}{4}$ **c)** 5 **d)** -3.5

Practise

For help with questions 1 and 2, see Example 1.

1. Find the equation of a line with the given slope and passing through the given point, P.

a) $m = 1$, P(3, 5)

b) $m = -3$, P(0, -4)

c) $m = \frac{2}{3}$, P(-2, 6)

d) $m = -\frac{1}{2}$, P(5, -2)

e) $m = -\frac{4}{5}$, P(0, 0)

f) $m = 2$, P$\left(\frac{1}{2}, \frac{3}{4}\right)$

2. Find the equation of a line

a) with a slope of -3, passing through the origin

b) parallel to $y = \frac{2}{3}x + 5$, passing through $(4, -5)$

c) parallel to the x-axis, passing through $(3, -6)$

d) perpendicular to $y = -\frac{2}{5}x + 4$, passing through the origin

e) perpendicular to $x = -2$, passing through the point $(1, -3)$

f) perpendicular to $y = 4x - 3$, passing through the point $(-2, 7)$

Connect and Apply

For help with questions 3 and 4, see Example 2.

3. In Niagara-on-the-Lake, you can ride a horse-drawn carriage for a fixed price plus a variable amount that depends on the length of the trip. The variable cost is \$10/km and a 2.5-km trip costs \$40.

a) Determine the equation relating cost, C, in dollars, and distance, d, in kilometres.

b) Use your equation to find the cost of a 6.5-km ride.

c) Graph this relation.

d) Use the graph to find the cost of a 6.5-km ride.

Making Connections

You learned about first differences and their relationship with slope in 5.6 Connecting Variation, Slope, and First Differences.

4. Refer to question 3.

a) Copy and complete the table to solve the problem using a third method. Explain this method.

Distance (km)	Cost (\$)	First Differences
2.5	40	
3.5	50	10
4.5		
5.5		
6.5		

b) Use all three methods (equation, graph, table) to determine how far you could travel in the horse-drawn carriage for \$100.

c) Use each method to determine the cost of a 5.8-km ride.

d) Describe at least one advantage and one disadvantage to each method of solution.

For help with questions 5 and 6, see Example 3.

5. Find an equation for the line parallel to $2x - 3y + 6 = 0$, with the same y-intercept as $y = 7x - 1$.

6. Find an equation for the line perpendicular to $4x - 5y = 20$ and sharing the same y-intercept.

7. **Chapter Problem** Jean's home city is one of the best designed in North America for traffic flow, Traffic lights are carefully programmed to keep cars moving. Some lanes on one-way streets change direction depending on the time of day. To find two more letters in the name of this city, find the x- and y-intercepts of the line that is perpendicular to $y = \frac{9}{8}x + 1$ and passes through the point $(18, -8)$.

8. Aki has been driving at an average speed of 80 km/h toward Ottawa for 3 h, when he sees the sign shown.
The equation relating distance and time is of the form $d = mt + b$.

a) What does the ordered pair (3, 300) mean?

b) The slope is $m = -80$. What does this value represent? Why is it negative?

c) Determine the value of b.

d) Write an equation relating distance and time.

e) Graph the relation. What is the meaning of the d-intercept?

f) How long will the trip to Ottawa take, in total?

g) Has Aki reached the halfway point of his trip yet? Explain.

9. **Use Technology** You can use *The Geometer's Sketchpad®* to solve the taxi problem in Example 2.

a) Follow these steps:
- Open a new sketch and display the grid.
- Create a new parameter and call it b.
- Create a new function and define it as $f(x) = 2x + b$.
- Plot the point (10, 25). Click and drag the two control points near the origin to adjust the scales and the position of the origin so that you can see this point.
- Manipulate the parameter b until the line passes through the point (10, 25).

b) Explain how this method works.

10. **Use Technology** A city taxi charges \$2.50/km plus a fixed cost. A 6-km taxi ride costs \$22. Use *The Geometer's Sketchpad®* to find

a) the fixed cost

b) the equation relating cost, C, in dollars, and distance, d, in kilometres

c) Find the equation using another method to check your results.

Extend

11. Refer to question 8. Suppose that, when Aki sees the sign, he increases his driving speed to 100 km/h.

a) Construct a graph to model Aki's trip.

b) How would your answers to parts f) and g) change?

c) Explain how you solved this problem.